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AGAINST PROBABILISTIC MEASURES OF COHERENCE

ABSTRACT. It is shown that the probabilistic theories of coherence proposed up to now produce a number of counter-intuitive results. The last section provides some reasons for believing that *no* probabilistic measure will ever be able to adequately capture coherence. First, there can be no function whose arguments are nothing but tuples of probabilities, and which assigns different values to pairs of propositions $\{A, B\}$ and $\{A, C\}$ if A implies both B and C, or their negations, and if $P(B) = P(C)$. But such sets may indeed differ in their degree of coherence. Second, coherence is sensitive to explanatory relations between the propositions in question. Explanation, however, can hardly be captured solely in terms of probability.

Why does the proposition 'Tweety is a bird' fit 'Tweety has wings' much better than 'Tweety cannot fly'? Because, one might argue, the probability that Tweety has wings, given that it is a bird, strongly exceeds the probability that Tweety cannot fly, given that it is a bird. Presumably, such considerations have led some philosophers to develop *probabilistic* measures of coherence. Tomoji Shogenji, Erik Olsson, Branden Fitelson, and Igor Douven and Wouter Meijs have presented functions which take as input certain probabilities relating to the propositions in question to calculate from them a number which is supposed to represent their degree of coherence.[1]

In Sections 1–5, I point out some difficulties with these proposals as well as with Luc Bovens and Stephan Hartmann's coherence quasi-ordering. However, even if these specific accounts prove to be deficient, this does certainly not mean that the notion of a probabilistic measure of coherence is *fundamentally* wrong. After all, this research project is still in its infancy, and it thus seems justifiable to adopt a 'don't worry it'll sort itself out' attitude. In opposition to this attitude, the last section provides some grounds for believing that *no* probabilistic function will ever be able to adequately capture coherence.

1. SHOGENJI'S MEASURE

According to Shogenji (1999, p. 240), the coherence of a system of propositions $\{A_1, \ldots, A_n\}$ is to be determined as follows:

$$C_S(A_1, \ldots, A_n) = \frac{P(A_1 \& \ldots \& A_n)}{P(A_1) \times \cdots \times P(A_n)}$$

$P(A_1) \times \cdots \times P(A_n)$ is the probability the conjunction $A_1 \& \ldots \& A_n$ would have if its constituents were statistically independent. Shogenji's formula thus tells us to which extent the actual probability that the propositions are true together deviates from the probability they would have if they were statistically independent. If the C_S-value is greater than 1, the set is coherent; if it is smaller than 1, the statements do not fit together, where 0 means that they are maximally incoherent.

Note that all probabilistic measures have to confront the question of whether it is possible to determine the relevant probabilities in a satisfactory way when we are faced with concrete problems. For example, in order to apply Shogenji's proposal, we need the prior probabilities $P(A_1)$ to $P(A_n)$. But if one of the propositions at issue is a newly developed scientific theory, how to fix its probability? We might often be able to confer on it a probability in the Bayesian sense of a subjective degree of confidence; but many epistemologists are not satisfied with such an approach because it smacks of an 'it's all at your discretion' stance. However, I will not follow this train of thought and would rather make concessions to supporters of probabilistic epistemology by taking into account only cases where the relevant probabilities, or ratios thereof, are unproblematic. If a probabilistic measure of coherence fails here already because it leads to inadequate results, the question of its applicability to other cases may be considered to be of secondary importance.

Shogenji's measure is counter-intuitive for a number of reasons. First, let there be ten equiprobable suspects for a murder. All of them previously committed at least one crime, two a robbery, two pickpocketing, and the remaining six both crimes. There is thus a substantial overlap: of the total of eight suspects who committed a robbery, six were also involved in pickpocketing, and conversely. On account of this strong coincidence, I see no reason why the pair of propositions

(M1) The murderer committed a robbery.
(M2) The murderer committed pickpocketing.

should be regarded incoherent. But in the light of Shogenji's proposal:

$$C_S(\mathrm{M1}, \mathrm{M2}) = \frac{P(\mathrm{M1}\,\&\,\mathrm{M2})}{P(\mathrm{M1}) \times P(\mathrm{M2})} = \frac{0.6}{0.8 \times 0.8} < 1,$$

which would mean that these assumptions do not fit together.

Generally, when we are confronted with two statements which cannot both be false, Shogenji's function assigns them a coherence value of 1 at most. For if $\neg A$ implies B, then $\neg A$ is logically equivalent to $\neg A$ & B so that:

$$\begin{aligned} P(A) &= 1 - P(\neg A) = 1 - P(\neg A\,\&\,B) \\ &\geq 1 - \frac{P(\neg A\,\&\,B)}{P(B)} = 1 - P(\neg A|B) = P(A|B) = \frac{P(A\,\&\,B)}{P(B)} \end{aligned}$$

Hence:

$$P(A\,\&\,B) \leq P(A) \times P(B)$$

But the fact that one of the assumptions in question must be true does not rule out that they cohere. In a nutshell, Shogenji's measure does not adequately treat subcontraries (cf. Siebel, 2004).

Second, Ken Akiba (2000, p. 357) has claimed that it is also on the wrong track with respect to two-member cases in which one of the propositions implies the other. If B logically follows from A, $P(A$ & $B)$ is identical with $P(A)$, entailing that $C_S(A, B) = 1/P(B)$. Thus, statements of this type would be the more coherent the less likely consequence B is. But the following example seems to suggest that this is a mistake (the die is, of course, meant to be fair):

(D1) The die will come up 2.
(D2) The die will come up 2 or 4.
(D3) The die will come up 2, 4 or 6.

D2, so Akiba says, dovetails with D1 just as much as D3 does because both propositions are logical consequences of D1. According to Shogenji's function, however, the coherence of {D1, D2} is 3, whereas {D1, D3} is merely coherent to degree 2.

In order to show that such systems do differ in coherence, Shogenji (2001, p. 148) refers us to a similar example. A fossil is dated by three measurements. The results are:

(F1) The fossil is 64–66 million years old.
(F2) The fossil is 63–67 million years old.
(F3) The fossil is more than 10 years old.

F1 implies both F2 and F3, and {F1, F2} is more coherent according to Shogenji's formula than {F1, F3} because $P(\text{F2}) < P(\text{F3})$. But this appears to be all right because, intuitively, F1 and F2 fit together better than F1 and F3. After all, '64--66 million years' is, in a natural sense, closer to '63--67 million years' than to 'more than 10 years' because the latter leaves open many more alternatives which are excluded by '64--66 million years'. For example, F3, but neither F1 nor F2, would be true if the fossil was 20 million years old. The same may be said to hold in the die example. The assumption that the die will come up 2 or 4 permits only one alternative not covered by D1, namely 4. On the other hand, the weaker assumption that the die will come up 2, 4 or 6 includes two possibilities running counter to D1. It thus appears reasonable to maintain that {D1, D2} registers a greater coherence than {D1, D3}.

So far, so good. But what about examples where the logical consequences comprise only *one* alternative to the proposition they follow from? Suppose a physicist cannot recall the voltage of a power source. She only knows that it is 1 V or 2 V or ... 50 V. She asks her assistants, and their answers are:

(V1) The voltage is 1 V.
(V2) The voltage is 1 or 2 V.
(V3) The voltage is 1 or 50 V.

Since a voltage of 2 V is as likely as a voltage of 50 V, V2's probability does not differ from V3's. Shogenji is thus committed to the claim that V1 dovetails with V3 just as much as with V2: $C_S(\text{V1, V3}) = 1/P(\text{V3}) = 1/P(\text{V2}) = C_S(\text{V1, V2})$.

But this is not the way in which our physicist will appraise the coherence of these pairs. It would rather appear that V1 fits V2 more than V3 because V2's alternative is much closer to V1 than V3's alternative. If V1 is false because the voltage is 2 V, there is at least a close proximity to V1. But if it is false because the voltage is 50 V, then the truth is miles away from V1. Shogenji's purely probabilistic approach overlooks that coherence is also sensitive to such distances between numerical values, thereby ignoring an aspect which is highly important for scientists.

Third, assume again that the probability for 2 V is the same as for 50 V. This time, however, the statements put forward by the physicist's assistants cannot be true together:

(V4) The voltage is 1 V.

(V5) The voltage is 2 V.
(V6) The voltage is 50 V.

On account of the inherent inconsistency, I am ready to grant that both {V4, V5} and {V4, V6} are *in*coherent. Nevertheless, the former set is *less* incoherent than the latter because 1 V is closer to 2 V than to 50 V. Presumably, our physicist will therefore disregard the possibility that the voltage is 50 V and assume that it is close to what the first two assistants have said. By applying Shogenji's measure, however, {V4, V5} and {V4, V6} would turn out to be incoherent to the same degree. Since V4 implies ¬V5 as well as ¬V6, $P(\text{V4} \,\&\, \text{V5}) = P(\text{V4} \,\&\, \text{V6}) = 0$. Hence, $C_S(\text{V4, V5})$ and $C_S(\text{V4, V6})$ are also 0, which means that these sets are indistinguishable with respect to coherence because they are both maximally incoherent.

More generally, I do not deny that inconsistencies have a negative *influence* on coherence. It might even be the case that propositions are coherent only if they are consistent (cf. BonJour 1985, p. 95). But this does not entail that contrary propositions make a system incoherent *to the maximum*. There may be further factors present which have a positive impact and thus partly compensate for inconsistencies. A closer proximity of numerical values is one such factor. By rendering sets of contrary statements maximally incoherent, Shogenji does not take into account this aspect of coherence, although it plays a significant role in scientific thinking.

Finally, Shogenji's measure entails that, by including the conjunction or disjunction of the propositions in question, coherence is usually increased. In the normal course of events, $P(A \,\&\, B)$ and $P(A \vee B)$ are smaller than 1. Moreover, both $A \,\&\, B \,\&\, (A \,\&\, B)$ and $A \,\&\, B \,\&(A \vee B)$ can be reduced to $A \,\&\, B$. Therefore:

$$\begin{aligned}
C_S(A, B, A \,\&\, B) &= \frac{P(A \,\&\, B)}{P(A) \times P(B) \times P(A \,\&\, B)} = \frac{C_S(A \,\&\, B)}{P(A \,\&\, B)} \\
&> C_S(A, B) \\
C_S(A, B, A \vee B) &= \frac{P(A \,\&\, B)}{P(A) \times P(B) \times P(A \vee B)} = \frac{C_S(A \,\&\, B)}{P(A \vee B)} \\
&> C_S(A, B)
\end{aligned}$$

But this is not acceptable as a general rule. There might be scenarios in which the extended set can be taken to be more coherent. For example, let there be two witnesses, one of them asserting that the murderer is blonde and the second reporting that he spoke with a Scandinavian accent. These propositions fit together quite well; but if a third witness appears claiming that the culprit is a blonde person with a Scandinavian accent, the propositions put forward by all three could be taken to be even more coherent.

However, if we consider just one person who does not integrate new input but merely draws conclusions from what she already believes, the above inequalities seem to amount to a 'cut-prize offer'. For they allow one to increase the coherence of one's doxastic system too simply, viz., just by inferring trivial consequences.[2] Just imagine a physicist arguing as follows: 'I concede that the General Theory of Relativity and the theory of Quantum Mechanics do not really fit together. But we could, effortlessly and without committing ourselves to anything beyond what we are already committed to, weaken the incoherence, merely by adding the conjunction or the disjunction of these theories. And if we do both, our system is in an even better shape.' This is obviously ridiculous. In order to obtain more coherence, our physicist has to try a bit harder.[3]

2. OLSSON'S MEASURE

Olsson (2002, p. 250) tentatively suggests a function quite similar to Shogenji's:

$$C_O(A_1, \ldots, A_n) = \frac{P(A_1 \,\&\, \ldots \,\&\, A_n)}{P(A_1 \vee \cdots \vee A_n)}$$

The idea here is: the closer the probability of *each* proposition being true is to the probability that *at least one* of them is true, the more they cohere. The values of this function range from 0 to 1, making clear at least that 0 represents maximal incoherence and 1 maximal coherence. There is no universal threshold, however, where incoherence passes into coherence. In particular, a value above (below) 0.5 must not be interpreted as meaning that the set is coherent (incoherent).

Although the lack of a neutral point is irrelevant to the following objections, it ensures that Olsson's measure is immune to the problem *subcontraries* pose for Shogenji's account. Remember the murder example. Since one of the statements must be true, $P(\mathrm{M1} \vee \mathrm{M2}) = 1$ and thus $C_O(\mathrm{M1}, \mathrm{M2}) = P(\mathrm{M1} \,\&\, \mathrm{M2}) = 0.6$. But just as this does not mean that these propositions are incoherent, so it does not mean they fit together. It merely shows that they are neither incoherent nor coherent *to the extreme*.

Moreover, in contrast to Shogenji's formula, Olsson's rules that adding the conjunction or disjunction of the statements in question does not affect coherence. $A \vee B \vee (A \& B)$, as well as $A \vee B \vee (A \vee B)$, are equivalent to $A \vee B$. Hence:

$$C_O(A, B, A \& B) = C_O(A, B, A \vee B) = \frac{P(A \& B)}{P(A \vee B)} = C_O(A, B)$$

However, including a *necessary truth* has the unwanted consequence of rendering the set less coherent (given that$P(A \vee B) < 1$). Since a disjunction of any proposition with a necessity N must be true, we get:

$$C_O(A, B, N) = \frac{P(A \& B \& N)}{P(A \vee B \vee N)} = P(A \& B) < \frac{P(A \& B)}{P(A \vee B)} \\ = C_O(A, B)$$

But what about *mathematical truths*? I presume Olsson does not want to say that, in order to stay as coherent as possible, we should avoid assimilating them to our belief system. Shogenji's account is doing better in this respect:

$$C_S(A, B, N) = \frac{P(A \& B \& N)}{P(A) \times P(B) \times P(N)} = \frac{P(A \& B)}{P(A) \times P(B)} = C_S(A, B)$$

As to contrary systems of assumptions, Olsson's measure, just as Shogenji's, cuts a bad figure. If A implies $\neg B$, then $C_O(A, B) = 0$. Hence, such sets would, without exception, be incoherent to the highest extent. But the second power source example (V4–V6) indicates that this is not true. Because of differences in the proximity of numerical values, contrary propositions may display different degrees of incoherence.

Furthermore, if both B and C are logical consequences of A, and if the probability of B does not differ from that of C, Olsson's formula rules that $\{A, B\}$ is coherent to the same extent as $\{A, C\}$. The disjunctions $A \vee B$ and $A \vee C$ are then equivalent to B and C, respectively. Therefore:

$$C_O(A, B) = \frac{P(A \& B)}{P(A \vee B)} = \frac{P(A)}{P(B)} = \frac{P(A)}{P(C)} = \frac{P(A \& C)}{P(A \vee C)} = C_O(A, C)$$

The first example of the power source (V1–V3) makes clear that this is a mistake.[4]

3. FITELSON'S MEASURE

In his article 'A Probabilistic Theory of Coherence' (2003), Fitelson offered a formula which he revised a bit in his online-paper 'Two Technical Corrections to My Coherence Measure' (2004). The starting point of his account is the idea that the degree of coherence depends on the degree of *support* the propositions provide for each other.[5] As to support, aka confirmation, Fitelson adopts Kemeny and Oppenheim's (1952) measure with a slight modification. If B implies A (and is not logically false), then he takes B to confirm A to the maximum degree: $F(A, B) = 1$. If B implies $\neg A$, the support is as small as it can be: $F(A, B) = -1$.[6] And if neither A nor its negation follow from B, the degree of confirmation arises from Kemeny and Oppenheim's original function:

$$F(A,B) = \frac{P(B|A) - P(B|\neg A)}{P(B|A) + P(B|\neg A)}$$

Given the required probabilities, we can calculate the extent to which each proposition in a set, and each conjunction of propositions, is supported by each remaining proposition and each conjunction of them. For example, for a trio of propositions we get 12 such numbers: $F(A, B)$, $F(A, C)$, $F(B, A)$, $F(B, C)$, $F(C, A)$, $F(C, B)$, $F(A, B \& C)$, $F(B, A \& C)$, $F(C, A \& B)$, $F(A \& B, C)$, $F(A \& C, B)$ and $F(B \& C, A)$. The degree of coherence is then defined as the straight average of these values, i.e., their sum divided by their number. Intuitively, coherence is thus identified with average confirmation. Like the values of the support function F, the numbers provided by this measure range from -1 to 1. A value above 0 means that the set is coherent, a value below 0 that it is incoherent.

Despite the initial appeal of Fitelson's recipe, it also produces counter-intuitive results. The first point is that the murder example proves as problematic for Fitelson as it does for Shogenji. Assume, again, that there are ten equiprobable suspects for a murder, where two previously committed a robbery, two pickpocketing and six both crimes. Owing to the sizeable overlap, the pair consisting of

(M1) The murderer committed a robbery.
(M2) The murderer committed pickpocketing.

does not seem to be incoherent. However, since each suspect was involved either in a robbery or in pickpocketing, $\neg$M1 implies M2.

Therefore, $P(\mathrm{M2}|\neg\mathrm{M1})$ and $P(\mathrm{M1}|\neg\mathrm{M2})$ are both equal to 1, which entails:

$$C_{\mathrm{F}}(\mathrm{M1},\mathrm{M2}) = \left(\frac{6/8-1}{6/8+1} + \frac{6/8-1}{6/8+1}\right)/2 < 0$$

Thus, Fitelson's measure leads to the claim that these propositions do not fit together.

This is only one instance of a general problem. Fitelson's measure is unable to cope with subcontraries because it does not allow them to be coherent, only neutral at most. For if $\neg A$ implies B, then:

$$P(A|B) - P(A|\neg B) = P(A|B) - 1 \leq 0$$
$$P(B|A) - P(B|\neg A) = P(B|A) - 1 \leq 0$$

And this means that $C_{\mathrm{F}}(A, B)$ does not exceed the threshold 0 above which coherence begins.[7]

The second point is related to the previous one. There is a simple way to convert any set of propositions into a system of which one statement must be true: just add a necessary truth. Hence, it should come as no surprise that Fitelson's measure has a consequence which resembles one of the deficiencies of Olsson's formula: by assimilating a necessity N to a pair of propositions which support each other to a sufficiently high degree, one *lowers* coherence. If none of the assumptions A or B implies the other one or its negation, $C_{\mathrm{F}}(A, B, N)$ is given by dividing the sum of the following values by 12:

$$F(A,B) = \frac{P(B|A) - P(B|\neg A)}{P(B|A) + P(B|\neg A)}$$
$$F(A,N) = \frac{P(N|A) - P(N|\neg A)}{P(N|A) + P(N|\neg A)} = \frac{1-1}{1+1} = 0$$
$$F(B,A) = \frac{P(A|B) - P(A|\neg B)}{P(A|B) + P(A|\neg B)}$$
$$F(B,N) = \frac{P(N|B) - P(N|\neg B)}{P(N|B) + P(N|\neg B)} = 0$$
$$F(N,A) = 1 (\text{because } N \text{ is implied by any proposition})$$
$$F(N,B) = 1$$
$$F(A, B \,\&\, N) = \frac{P(B \,\&\, N|A) - P(B \,\&\, N|\neg A)}{P(B \,\&\, N|A) + P(B \,\&\, N|\neg A)} = F(A,B)$$
$$F(B, A \,\&\, N) = \frac{P(A \,\&\, N|B) - P(A \,\&\, N|\neg B)}{P(A \,\&\, N|B) + P(A \,\&\, N|\neg B)} = F(B,A)$$

$$F(N, A \,\&\, B) = 1$$

$$F(A \,\&\, B, N) = \frac{P(N|A \,\&\, B) - P(N|\neg(A \,\&\, B))}{P(N|A \,\&\, B) + P(N|\neg(A \,\&\, B))} = 0$$

$$F(A \,\&\, N, B) = \frac{P(B|A \,\&\, N) - P(B|\neg(A \,\&\, N))}{P(B|A \,\&\, N) + P(B|\neg(A \,\&\, N))} = F(A, B)$$

$$F(B \,\&\, N, A) = \frac{P(A|B \,\&\, N) - P(A|\neg(B \,\&\, N))}{P(A|B \,\&\, N) + P(A|\neg(B \,\&\, N))} = F(B, A)$$

Now suppose that A and B confirm each other to such a degree that $F(A, B) + F(B, A) > 1$. Then:

$$\begin{aligned} C_{\mathrm{F}}(A, B, N) &= [3 \times F(A, B) + 3 \times F(B, A) + 3 \times 1 + 3 \times 0]/12 \\ &< [6 \times F(A, B) + 6 \times F(B, A)]/12 = C_{\mathrm{F}}(A, B) \end{aligned}$$

This is surprising in itself; but what is even more surprising is that it is possible to obtain the opposite result as well, namely if $F(A, B) + F(B, A)$ is smaller than 1. That is, the impact of necessities would depend on how much A and B support each other. This is a behaviour for which I see no good reason.

Note that I do not call into question the intuitive idea underlying Fitelson's proposal. The above-mentioned difficulties do not originate from the fact that he wants to capture coherence in terms of *confirmation*; they are rather due to his wish to get a grip on coherence by relying on nothing but *probability*. For in order to construct a measure of coherence which is both probabilistic and sensitive to confirmation, he has to make use of a probabilistic account of confirmation. It is this translation into the language of probability that is causing the problem.

There is disagreement among advocates of probabilistic epistemology on the quantitative question of how to measure the degree to which B supports A. Some rely on the difference $P(A|B) - P(A)$, others prefer the ratio $P(A|B)/P(A)$ or a logarithm thereof, Fitelson adopts Kemeny and Oppenheim's function, and there are further proposals.[8] But when it comes to the qualitative question under which conditions B provides any confirmation for A at all, they unanimously rely on the so-called relevance criterion:

> B confirms A iff $P(A|B) > P(A)$; B disconfirms A iff $P(A|B) < P(A)$; and B is confirmationally irrelevant to A iff $P(A|B) = P(A)$[9]

From this principle, we are allowed to infer that B undermines A if it is implied by its negation (provided B does not have a prior probability of 1). For then $P(\neg A \;\&\; B) = P(\neg A)$ so that:

$$P(A|B) = 1 - P(\neg A|B) = 1 - \frac{P(\neg A \,\&\, B)}{P(B)} = 1 - \frac{P(\neg A)}{P(B)}$$
$$< 1 - P(\neg A) = P(A)$$

A proponent of the relevance criterion is thus committed to the claim that evidence cannot speak in favour of a hypothesis if they are subcontraries. This result in itself might already arouse some suspicion. Instead of jumping to conclusions, however, let us see what happens if it is considered in connection with the starting point of Fitelson's measure of coherence.

Fitelson assumes that coherence is sensitive to support. This is perfectly reasonable. If we look at how scientists conceive of confirmation and coherence, a particular connection between them is quite obvious: a hypothesis fits the experimental data only if they support it. Or put the other way round, if the data disconfirm the hypothesis, it does not cohere with them. Assume a chemist carries out an experiment on Boyle's Law, which states that, at a constant temperature, the pressure of a given quantity of gas is inversely proportional to its volume. Before the chemist reduces the volume of the gas by half, the barometer measures a pressure of 1 bar; afterwards its reading is 3 bars. Since Boyle's Law predicts 2 bars, the chemist will thus take her experimental result to undermine the former (if she thinks that the barometer is reliable), inferring from this that Boyle's Law does not dovetail with her observation. Thus, scientifically pertinent conceptions of confirmation and coherence should respect the following bridge principle:

$$\text{If } B \text{ disconfirms } A, \text{ then } \{A, B\} \text{ is incoherent.}$$

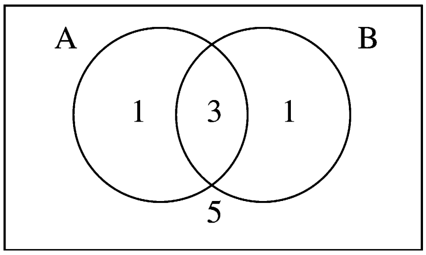
The relevance criterion, however, entails that B speaks against A if one of them must be true and $P(B)$ is smaller than 1. Hence, together with the bridge principle, it rules:

$$\text{If } A \text{ and } B \text{ are subcontraries and } P(B) < 1,$$
$$\text{then } \{A, B\} \text{ is incoherent.}$$

But the murder example (M1–M2) indicates that this is questionable. The given propositions are subcontraries with a probability below 1. Nevertheless, they appear to fit together.

The moral here is that the relevance criterion is dubious. It is therefore no wonder that a measure of coherence based on it will run into problems. As mentioned, the reason for the inadequate treatment of subcontrary propositions does not lie in Fitelson's idea that coherence can be spelled out in terms of confirmation. The bridge principle makes clear that these notions are indeed connected. Rather, the problem lies in Fitelson's probabilistic orientation, i.e., his presumption that confirmation, and thereby coherence, can be given a purely probabilistic shape.

A third problem is that Fitelson's measure agrees with Shogenji's in permitting the coherence to be increased by adding the conjunction of the propositions at issue. Consider the following distribution:



Since this distribution is symmetrical, and since $A \,\&\, B$ implies both A and B and is implied by A and B together:

$$F(A, B) = F(B, A) = \frac{3/4 - 1/6}{3/4 + 1/6} \approx 0.64$$

$$\begin{aligned} &F(A, (A \,\&\, B)) = F(B, (A \,\&\, B)) = F(A, B \,\&\, (A \,\&\, B)) \\ &\quad = F(B, A \,\&\, (A \,\&\, B)) = F((A \,\&\, B), A \,\&\, B) \\ &\quad = F(A \,\&\, B, (A \,\&\, B)) = 1 \\ &F((A \,\&\, B), A) = F((A \,\&\, B), B) = F(A \,\&\, (A \,\&\, B), B) \\ &\quad = F(B \,\&\, (A \,\&\, B), A) = \frac{1 - 1/7}{1 + 1/7} = 0.75 \end{aligned}$$

Hence:

$$\begin{aligned} C_{\mathrm{F}}(A, B, A \,\&\, B) &\approx [2 \times 0.64 + 6 \times 1 + 4 \times 0.75]/12 \approx 0.86 \\ &> [2 \times 0.64]/2 = 0.64 \approx C_{\mathrm{F}}(A, B) \end{aligned}$$

The same happens when we add the disjunction $A \vee B$. Since both A and B entail it, and since we have a symmetrical distribution:

$$\begin{aligned} F(A,B) &= F(A, B \,\&\, (A \vee B)) = F(A \,\&\, (A \vee B), B) = F(B, A) \\ &= F(B, A \,\&\, (A \vee B)) = F(B \,\&\, (A \vee B), A) \approx 0.64 \end{aligned}$$

$$F(A \vee B, A) = F(A \vee B, B) = F(A \vee B, A \,\&\, B) = 1$$

$$F(A, A \vee B) = F(B, A \vee B) = \frac{1 - 1/6}{1 + 1/6} \approx 0.71$$

$$F(A \,\&\, B, A \vee B) = \frac{1 - 2/7}{1 + 2/7} \approx 0.56$$

And so:

$$\begin{aligned} C_{\mathrm{F}}(A, B, A \vee B) &\approx [6 \times 0.64 + 3 \times 1 + 2 \times 0.71 + 0.56]/12 \\ &\approx 0.74 > [2 \times 0.64]/2 = 0.64 \approx C_{\mathrm{F}}(A, B) \end{aligned}$$

As already stated in connection with Shogenji's proposal, this would mean that the coherence of a system can be improved simply by drawing trivial conclusions.

Fourth, Fitelson's measure fares no better than Shogenji's and Olsson's when we look at pairs of contrary statements. If A and B exclude each other, both $F(A, B)$ and $F(B, A)$ are -1, which entails that $C_{\mathrm{F}}(A, B)$ is also -1. That is, Fitelson's formula puts on a par all two-member systems of inconsistent assumptions, namely as being incoherent to the highest degree. But remember the example of the physicist who wants to know the voltage of a power source. The prior probability of that voltage being 2 V is identical with the prior probability of its being 50 V. She asks her assistants, and their answers are:

(V4) The voltage is 1 V.
(V5) The voltage is 2 V.
(V6) The voltage is 50 V.

{V4, V5} is less incoherent than {V4, V6} because 1 V is closer to 2 V than to 50 V.

Fifth, as an advantage of his measure over Shogenji's, Fitelson (2003, p. 179) points out that, if A implies B, the degree of coherence depends not only on the probability of B but also on the statistical relationship between A and B:

$$C_{\mathrm{F}}(A,B)=\left(\frac{1-P(B|\neg A)}{1+P(B|\neg A)}+\frac{P(A|B)-0}{P(A|B)+0}\right)/2$$
$$=\left(\frac{1-P(B|\neg A)}{1+P(B|\neg A)}+\frac{1+P(B|\neg A)}{1+P(B|\neg A)}\right)/2=\frac{1}{1+P(B|\neg A)}$$

Although it might be reasonable to let the relationship between A and B play a role here, I do not believe that Fitelson's specific account is on the right track. Take the other power source example, where a voltage of 2 V is again as probable as a voltage of 50 V:

(V1) The voltage is 1 V.
(V2) The voltage is 1 or 2 V.
(V3) The voltage is 1 or 50 V.

Fitelson is obliged to claim that these pairs possess the same coherence. Since both ¬V2 and ¬V3 imply ¬V1, $P(\neg V1|\neg V2) = P(\neg V1|\neg V3) = 1$. Hence, since $P(\neg V2) = P(\neg V3)$:

$$C_{\mathrm{F}}(\mathrm{V1},\mathrm{V2})=\frac{1}{1+1-P(\neg\mathrm{V2}|\neg\mathrm{V1})}=\frac{1}{2-P(\neg\mathrm{V2})\times\frac{P(\neg\mathrm{V1}|\neg\mathrm{V2})}{P(\neg\mathrm{V1})}}$$
$$=\frac{1}{2-\frac{P(\neg\mathrm{V2})}{P(\neg\mathrm{V1})}}=\frac{1}{2-\frac{P(\neg\mathrm{V3})}{P(\neg\mathrm{V1})}}=C_{\mathrm{F}}(\mathrm{V1},\mathrm{V3})$$

But {V1, V2} is endowed with a greater coherence than {V1, V3} because V2's alternative to V1 is numerically closer to V1 than V3's alternative.[10]

4. DOUVEN AND MEIJS'S MEASURE

Douven and Meijs's (2006, sect. 3) proposal is quite similar to Fitelson's. They diverge merely in terms of choosing the difference measure of confirmation instead of (a modified variant of) Kemeny and Oppenheim's formula. Thus, they take the coherence of a two-member set as given by:

$$C_{\mathrm{DM}}(A,B)=[P(A|B)-P(A)+P(B|A)-P(B)]/2$$

The values of this function range from −1 to 1. Douven and Meijs do not explicitly distinguish a threshold above which coherence begins. But their account suggests that, like Fitelson, they have 0 in mind. For it appears that they consider pairs of propositions which neither support nor undermine each other to be neither coherent nor incoherent. Now, in terms of the relevance criterion, A and B are confirmationally irrelevant to each other just in case $P(A|B) = P(A)$, which entails that $P(B|A) = P(B)$. But then $P(A|B) - P(A) = P(B|A) - P(B) = 0$; and thus $C_{\mathrm{DM}}(A, B)$ is also 0. It is therefore reasonable to assume that Douven and Meijs interpret a value of 0 as meaning that the pair is neither coherent nor incoherent. Analogously, if there is mutual disconfirmation between the propositions – viz., $P(A|B) < P(A)$ so that also $P(B|A) < P(B)$ – then $C_{\mathrm{DM}}(A, B) < 0$, suggesting that a value smaller than 0 stands for incoherence.

It should be noted that Douven and Meijs (2006, sect. 5.1) restrict their measure to sets of propositions which are pairwise logically independent. Thereby they immunise it against a number of objections, including the ones following below. However, since such a scope restriction, in view of counter-examples, smacks of an 'easy way out', I take the liberty of presenting some consequences of the unqualified variant of their proposal.

The first difficulty is that adding a necessity would *decrease* coherence. For $C_{\mathrm{DM}}(A, B, N)$ is identical with

$$\begin{aligned}[]&[P(A|B) - P(A) + P(A|N) - P(A) + P(B|A) - P(B) + P(B|N)\\ &\quad - P(B) + P(N|A) - P(N) + P(N|B) - P(N) + P(A|B \,\&\, N)\\ &\quad - P(A) + P(B|A \,\&\, N) - P(B) + P(N|A \,\&\, B) - P(N)\\ &\quad + P(A \,\&\, B|N) - P(A \,\&\, B) + P(A \,\&\, N|B) - P(A \,\&\, N)\\ &\quad + P(B \,\&\, N|A) - P(B \,\&\, N)]/12\end{aligned}$$

But this reduces to

$$3 \times [P(A|B) - P(A) + P(B|A) - P(B)]/12,$$

which is obviously smaller than

$$[P(A|B) - P(A) + P(B|A) - P(B)]/2$$

Second, by including the conjunction or disjunction of A and B, we could easily *increase* coherence. By taking the distribution used as a test case for Fitelson's measure, we get $C_{\mathrm{DM}}(A, B) = 0.35$, $C_{\mathrm{DM}}(A, B, A \,\&\, B) \approx 0.53$ and $C_{\mathrm{DM}}(A, B, A \vee B) \approx 0.39$. (I spare the reader the calculations.)

Third and fourth, {A, B} and {A, C} would be (in)coherent to the same degree if A implies the equiprobable statements B and C or their negations. In the former case:

$$C_{\mathrm{DM}}(A,B) = [P(A) \times P(B|A)/P(B) - P(A) + P(B|A) - P(B)]/2$$
$$= [P(A)/P(C) - P(A) + 1 - P(C)]/2 = C_{\mathrm{DM}}(A,C)$$

In the latter case:

$$C_{\mathrm{DM}}(A,B) = [0 - P(A) + 0 - P(B)]/2$$
$$= [0 - P(A) + 0 - P(C)]/2 = C_{\mathrm{DM}}(A,C)$$

The power source examples (V1–V3) and (V4–V6) speak against these equations.

The fifth and final difficulty of Douven and Meijs's measure is that pairs of subcontrary propositions would not fit together, even if there was a high amount of overlap. For if $\neg A$ entails B, $P(A|B) < P(A)$ and $P(B|A) < P(B)$ so that $C_{\mathrm{DM}}(A,B) < 0$. But recall the murder example (M1–M2).

5. BOVENS AND HARTMANN'S QUASI-ORDERING

Although the main topic of this paper is probabilistic *measures* of coherence, I do not want to ignore Bovens and Hartmann's *quasi-ordering*. In their book *Bayesian Epistemology* (2003b, chs. 1f.), they argue that the following function enables us to determine the relative coherence of two sets of propositions (see also Bovens and Hartmann 2003a, Sects. 4f.):

$$C_r(A_1, \ldots A_n) = \frac{a_0 + (1 - a_0)(1 - r)^n}{\sum_{i=0}^{n} a_i (1 - r)^i},$$

where a_i is the probability that i of the n statements are false, and r is the reliability of the statements' sources. (The sources are supposed to be independent in a specific way; and they are partially reliable such that, on the definition of r, $0 < r < 1$.) To be sure, C_r does not represent the propositions' degree of coherence because it is functionally dependent on the credibility of their sources. Bovens and Hartmann's claim is rather that supplementing this formula with a simple assumption makes it possible to compare systems of statements with respect to coherence:

> S_1 is at least as coherent as S_2 iff for all values of r, $C_r(S_1) \geq C_r(S_2)$.

Thus, if the C_r-values for S_1 are, for all degrees of partial reliability, always greater (smaller) than the corresponding values for S_2, then S_1 is more (less) coherent than S_2.

Bovens and Hartmann present a number of examples in order to show that their model is in a better position than those of Shogenji, Olsson and Fitelson. As far as these examples are concerned, this is true. However, they did not examine what happens when a necessary truth N is added to a system of propositions. Let a_i be the probability that i statements in the two-member set $\{A, B\}$ are false, and b_i the probability that i propositions in the extended set $\{A, B, N\}$ are false. Then:

$$\begin{aligned}
b_0 &= P(A \,\&\, B \,\&\, N) = P(A \,\&\, B) = a_0 \\
b_1 &= P(A \,\&\, B \,\&\, \neg N) + P(A \,\&\, \neg B \,\&\, N) + P(\neg A \,\&\, B \,\&\, N) \\
&= P(A \,\&\, \neg B) + P(\neg A \,\&\, B) = a_1 \\
b_2 &= P(A \,\&\, \neg B \,\&\, \neg N) + P(\neg A \,\&\, \neg B \,\&\, N) + P(\neg A \,\&\, B \,\&\, \neg N) \\
&= P(\neg A \,\&\, \neg B) = a_2 \\
b_3 &= P(\neg A \,\&\, \neg B \,\&\, \neg N) = 0
\end{aligned}$$

Therefore, $C_r(A, B, N)$ reads:

$$\frac{b_0 + (1 - b_0)(1 - r)^3}{b_0 + b_1(1 - r) + b_2(1 - r)^2 + b_3(1 - r)^3} = \frac{a_0 + (1 - a_0)(1 - r)^3}{a_0 + a_1(1 - r) + a_2(1 - r)^2},$$

while $C_r(A, B)$ is:

$$\frac{a_0 + (1 - a_0)(1 - r)^2}{a_0 + a_1(1 - r) + a_2(1 - r)^2}$$

The denominators of these fractions are identical. Since $0 < (1-r) < 1$, $(1-r)^3 < (1-r)^2$. Hence, if a_0, i.e., $P(A \,\&\, B)$, is smaller than 1 (which it usually will be), then $C_r(A, B, N)$ is smaller than $C_r(A, B)$ for all values of the reliability parameter r. This means, according to Bovens and Hartmann's proposal, that enriching a pair of such statements with a necessary truth lowers coherence. Their model is thus subject to the same difficulty as Olsson's. It is hardly acceptable that learning a truth of mathematics makes one's doxastic system less coherent.

Second, like Shogenji's account, Bovens and Hartmann's entails that assimilating the conjunction of two propositions is a reasonable method for increasing coherence. Again, a_i stands for the probability that i statements from pair $\{A, B\}$ are false, while b_i is the probability that i propositions in the extended set $\{A, B, A \,\&\, B\}$ are false. Then:

$$\begin{aligned}
b_0 &= P(A \,\&\, B \,\&\, (A \,\&\, B)) = P(A \,\&\, B) = a_0 \\
b_1 &= P(A \,\&\, B \,\&\, \neg(A \,\&\, B)) + P(A \,\&\, \neg B \,\&\, (A \,\&\, B)) \\
&\quad + P(\neg A \,\&\, B \,\&\, (A \,\&\, B)) = 0 \\
b_2 &= P(A \,\&\, \neg B \,\&\, \neg(A \,\&\, B)) + P(\neg A \,\&\, \neg B \,\&\, (A \,\&\, B)) \\
&\quad + P(\neg A \,\&\, B \,\&\, \neg(A \,\&\, B)) = P(A \,\&\, \neg B) \\
&\quad + P(\neg A \,\&\, B) = a_1 \\
b_3 &= P(\neg A \,\&\, \neg B \,\&\, \neg(A \,\&\, B)) = P(\neg A \,\&\, \neg B) = a_2
\end{aligned}$$

Since $1 - a_0 = a_1 + a_2$, this entails:

$$\begin{aligned}
C_r(A, B, A \,\&\, B) &= \frac{b_0 + (1 - b_0)(1 - r)^3}{b_0 + b_1(1 - r) + b_2(1 - r)^2 + b_3(1 - r)^3} \\
&= \frac{a_0 + a_1(1 - r)^3 + a_2(1 - r)^3}{a_0 + a_1(1 - r)^2 + a_2(1 - r)^3},
\end{aligned}$$

While

$$C_r(A, B) = \frac{a_0 + a_1(1 - r)^2 + a_2(1 - r)^2}{a_0 + a_1(1 - r) + a_2(1 - r)_2}$$

$C_r(A, B, A\&B)$ is greater than $C_r(A, B)$ if $\frac{C_r(A, B, A \,\&\, B)}{C_r(A, B)} > 1$, that is, if

$$\left[a_0 + a_1(1 - r)^2 + a_2(1 - r)^3\right] \times \left[a_0 + a_1(1 - r)^2 + a_2(1 - r)^2\right]$$

is greater than

$$\left[a_0 + a_1(1 - r)^2 + a_2(1 - r)^3\right] \times \left[a_0 + a_1(1 - r)^2 + a_2(1 - r)^2\right]$$

The former is

$$\begin{aligned}
&a_0^2 + \underline{a_0 a_1 (1 - r)} + a_0 a_2 (1 - r)^2 + \underline{a_0 a_1 (1 - r)^3} + a_1^2 (1 - r)^4 \\
&+ a_1 a_2 (1 - r)^5 + a_0 a_2 (1 - r)^3 + a_1 a_2 (1 - r)^4 + a_2^2 (1 - r)^5
\end{aligned}$$

The latter is

$$\begin{aligned}
&a_0^2 + \underline{a_0 a_1 (1 - r)^2} + a_0 a_2 (1 - r)^2 + \underline{a_0 a_1 (1 - r)^2} + a_1^2 (1 - r)^4 \\
&+ a_1 a_2 (1 - r)^4 + a_0 a_2 (1 - r)^3 + a_1 a_2 (1 - r)^5 + a_2^2 (1 - r)^5
\end{aligned}$$

Since these sums differ only in the underlined addends, the former is greater than the latter if

$$(1-r)+(1-r)^3-(1-r)^2-(1-r)^2>0$$

The left hand side of this equation is identical with

$$[(1-r)^2-(1-r)]\times[(1-r)-1]$$

Since $0<(1-r)<1$, $(1-r)^2<(1-r)<1$, so that both of these multiplicands are negative and their product is positive. Therefore, if neither a_0 nor a_1 is 0, i.e., $A\,\&\,B, A\,\&\,\neg B$ and $\neg A\,\&\,B$ do not have zero probability, then $C_r(A,B,A\,\&\,B)>C_r(A,B)$ for all values of the reliability parameter r.

An analogous derivation shows that Bovens and Hartmann's proposal amounts to the claim that adding the *disjunction* of two propositions makes the system *less* coherent. They should then approve of the following instruction: if you are interested in maximising coherence, infer from two propositions that their conjunction is true, but do not infer that at least one of them is true. I cannot see any justification for taking that stance.Third, like all the other models, Bovens and Hartmann's does not properly capture the power source example (V1–V3). If both B and C follow from A, and if these consequences possess the same probability, then:

$$\begin{aligned}
P(A\,\&\,B) &= P(A) = P(A\,\&\,C)\\
P(A\,\&\,\neg B) &= 0 = P(A\,\&\,\neg C)\\
P(\neg A\,\&\,B) &= P(\neg(A\vee\neg B)) = 1-P(A\vee\neg B)\\
&= 1-(P(A)+P(\neg C)) = P(\neg A\,\&\,C)\\
P(\neg A\,\&\,\neg B) &= P(\neg B) = P(\neg C) = P(\neg A\,\&\,\neg C)
\end{aligned}$$

Therefore:

$$\begin{aligned}
C_r(A,B) &= \frac{P(A)+(1-P(A))(1-r)^2}{P(A)+P(\neg A\,\&\,B)(1-r)+P(\neg B)(1-r)^2}\\
&= \frac{P(A)+(1-P(A))(1-r)^2}{P(A)+P(\neg A\,\&\,C)(1-r)+P(\neg C)(1-r)^2} = C_r(A,C)
\end{aligned}$$

That is, in the light of Bovens and Hartmann's account, pairs of this fashion cannot differ in coherence. Due to the greater proximity of the numerical values, however, 'The voltage is 1 V' fits 'The voltage is 1 or 2 V' more than 'The voltage is 1 or 50 V', even if the latter propositions are equally likely.

A similar problem arises in the case of contrary statements. If A implies both $\neg B$ and $\neg C$, and if $P(B) = P(C)$, then it follows:

$$\begin{aligned} P(A \,\&\, B) &= 0 = P(A \,\&\, C) \\ P(A \,\&\, \neg B) &= P(A) = P(A \,\&\, \neg C) \\ P(\neg A \,\&\, B) &= P(B) = P(C) = P(\neg A \,\&\, C) \\ P(\neg A \,\&\, \neg B) &= P(\neg(A \vee B)) = 1 - P(A \vee B) \\ &= 1 - (P(A) + P(B)) = 1 - (P(A) + P(C)) \\ &= P(\neg A \,\&\, \neg C) \end{aligned}$$

And thus:

$$\begin{aligned} C_r(A,B) &= \frac{(1-r)^2}{(P(A)+P(B))(1-r) + P(\neg A \,\&\, \neg B)(1-r)^2} \\ &= \frac{(1-r)^2}{(P(A)+P(C))(1-r) + P(\neg A \,\&\, \neg C)(1-r)^2} \\ &= C_r(A,C) \end{aligned}$$

But look at the example (V4–V6). Although neither 'The voltage is 1 V' and 'The voltage is 2 V' nor the former and 'The voltage is 50 V' fit together, the incoherence is weaker in the former case because 2 V is closer to 1 V.

6. SUMMARY OF THE OBJECTIONS

The following table summarises the objections presented in the previous sections. A minus sign means that the case at hand causes difficulties for the corresponding account while a plus sign means that it is adequately handled. Two boxes must be left empty. Neither

	Contrary propositions (V4–V6)	Subcontrary propositions (M1–M2)	Conjunctions and disjunctions	Logical consequences (V1–V3)	Necessary truths
Shogenji	–	–	–	–	+
Olsson	–		+	–	–
Fitelson	–	–	–	–	–
Douven & Meijs	–	–	–	–	–
Bovens & Hartmann	–		–	–	–

Olsson's nor Bovens and Hartmann's theories state that subcontrary propositions do not dovetail. The reason is, however, that their formulae are not connected with a specification of a numerical range standing for incoherence. I therefore deem a plus sign as misleading as a minus sign.

Regarding the inclusion of the conjunction or disjunction of two statements, I consider a *change* in coherence as problematic because, if a person just infers such a proposition from what she already believes, this does not seem to have an influence on the coherence of her doxastic system. Remember, however, that there are other cases where an increase in coherence can be attested. For example, let two witnesses report A and B, respectively, and then a third one A & B. Since intuitions vary, depending on the origins of the statements in question,[11] I do not wish to overemphasise this issue. None of the measures under consideration can do justice to *both* types of cases because they treat them in a uniform way, regardless of whether the conjunction was put forward by a further witness or merely inferred by someone from what she already accepted. But note also that this is no excuse for the behaviour of Bovens and Hartmann's quasi-ordering, according to which the conjunction *raises* coherence whereas the disjunction *lowers* it.

7. GENERAL OBJECTIONS

It has been argued that none of the probabilistic measures of coherence that have been put on the market withstands close scrutiny. This is, of course, far from proving that it is *impossible* to construct such a measure. 'Shogenji, Olsson, Fitelson, and Douven and Meijs went wrong, to be sure; but let us wait and see what the future holds in store', so one might think.

But note that some of the objections apply to *all* proposals. Among other things, none of them has been able to deal with the power source example (V1--V3). The downside is that this problem generalises. *Any* probabilistic measure, whatever it looks like in detail, has to assign the same degree of coherence to $\{A, B\}$ and $\{A, C\}$ if both B and C are logical consequences of A, and if these consequences possess the same probability. But this is not tenable because such pairs may very well differ in the degree to which they fit together.

A probabilistic measure consists in a function whose arguments are tuples of probabilities relating to the propositions in question.

For the two-member case $\{A, B\}$, these tuples might contain $P(A)$, $P(\neg B)$, $P(A \,\&\, B)$, $P(A \vee B)$, $P(A|B)$, $P(B|\neg A)$, and so on. Moreover, a function is an *unambiguous* relation. That is, by using the same argument, we obtain the same value. The problem then is: if a proposition A implies both B and C, and if $P(B) = P(C)$, then each and every probability relating to A and B is identical with the corresponding probability involving A and C. For the joint probability distributions for these sets of propositions will be identical.

Such distributions specify the probability of each possible combination of values for the given variables. In the case of a pair of statements, whose values are *true* and *false*, they determine the probabilities of $A \,\&\, B$, $A \,\&\, \neg B$, $\neg A \,\&\, B$ and $\neg A \,\&\, \neg B$. Moreover, given a joint probability distribution, any further probability about the domain is fixed because it can be calculated from the distribution.

Now, in connection with Bovens and Hartmann's quasi-ordering, we have seen already that, if B and C are equiprobable consequences of A, then $P(A \,\&\, B) = P(A \,\&\, C)$, $P(A \,\&\, \neg B) = P(A \,\&\, \neg C)$, $P(\neg A \,\&\, B) = P(\neg A \,\&\, C)$ and $P(\neg A \,\&\, \neg B) = P(\neg A \,\&\, \neg C)$. That is, the joint probability distributions for $\{A, B\}$ and $\{A, C\}$ are the same, entailing that there is no difference between the probabilities belonging to $\{A, B\}$ and the corresponding probabilities relating to $\{A, C\}$. Hence, in such a case the tuples entering our probabilistic coherence function are *identical*, *no matter which tuples of probabilities the function takes as arguments*. And since a function provides identical values for identical arguments, it will assign these sets the same coherence value. Consequently, probabilistic measures are not able to differentiate finely enough. After all, the power source example (V1--V3) has shown that there are cases of this type where differences in the proximity of numerical values lead to differences in coherence.

The same holds for contrary statements. In the section on Bovens and Hartmann's approach, it was also pointed out that the joint probability distributions for $\{A, B\}$ and $\{A, C\}$ are identical if A implies both $\neg B$ and $\neg C$ and if $P(B) = P(C)$. Thus, a probabilistic coherence function must necessarily turn out the same value for such pairs. But consider example (V4--V6). Even if a voltage of 2 V is as likely as a voltage of 50 V, I presume that a physicist would take the former to be less in disharmony with 'The power source has a voltage of 1 V' than the latter. If one puts on one's 'probabilistic glasses', one will not see this difference because they are too weak.

Aside from these direct arguments, there is a more indirect one. Laurence BonJour (1985, p. 98) has emphasised that "the coherence of a system [...] is enhanced by the presence of *explanatory* relations between its members".[12] The more of them can be established and the better the explanations are, the more the propositions fit together. For example, if hypothesis H_1 makes for an explanation for the data D_1 and D_2, whereas H_2 explains only D_1, then $\{H_1, D_1, D_2\}$ is, ceteris paribus, more coherent than $\{H_2, D_1, D_2\}$. And if H_1 explains D_1 better than D_2, then $\{H_1, D_1\}$ is, ceteris paribus, more coherent than $\{H_1, D_2\}$.

The crucial point then is that a proper theory of coherence must take such principles into account. That is, in order to gain control over *coherence* with probabilistic means, it is required that *explanation* be captured solely in terms of probability. But the proposals put forward so far give little cause for hope.

It is well known that Hempel's (1965) DN/IS model, according to which, roughly, H explains D if P(D|H) is close to 1, has failed. It is highly likely that Tom will not become pregnant, given that he regularly takes birth control pills; but this fact does not explain the former. Neither does Salmon's (1970) idea work, which is, again roughly, that H explains D if $P(\mathrm{D}|\mathrm{H}) > P(\mathrm{H})$ and H is not screened off from D by other putative explanantia. One of Achinstein's (1983, p. 168) counter-examples to Hempel's account proves that this is wrong. Suppose Susan swallows a pound of arsenic in order to commit suicide. Shortly after, however, she dies because she is run over by a bus. Then it is the collision with the bus, and not the arsenic, which explains her death. But let us assume that, in contrast to swallowing a pound of arsenic, such a collision does not always lead to death. Then $P(\text{death}|\text{arsenic}) > P(\text{death})$, and the collision does not screen off ingestion of arsenic from death because $P(\text{death}|\text{bus \& arsenic})$ is not identical with, but higher than, $P(\text{death}|\text{bus})$. Salmon's account thus rules that Susan's death can be explained by her swallowing arsenic.

'But', one may reply, 'this merely shows that Hempel and Salmon have failed. Why should we assume that any future account will do no better?' Here is the reason why I believe that this assumption is indeed warranted. Consider the following statements:

(D) The barometers in Hamburg fall.
(H2) There is a drop in atmospheric pressure in Hamburg & A drop in atmospheric pressure causes barometers to fall.
(H3) The barometers in Hamburg fall & (There is a drop in atmospheric pressure in Hamburg $\vee$ The barometers in Hamburg do not fall) & A drop in atmospheric pressure causes barometers to fall.

Both H2 and H3 entail D. But whereas H2 provides an explanation for D, H3 does not. After all, H3 would not imply D if it did not already include the latter; and an inference which essentially contains its conclusion as a premise can hardly be of explanatory value. One cannot explain an event by itself.

A probabilistic theory, however, will tell us that H3 explains D just as much as H2 does. For H3 is equivalent to H2, and thus $P(\mathrm{H3}) = P(\mathrm{H2})$. Moreover, since D is a consequence of both H2 and H3, $P(\mathrm{D} \,\&\, \mathrm{H2}) = P(\mathrm{H2})$ and $P(\mathrm{D} \,\&\, \mathrm{H3}) = P(\mathrm{H3})$. Therefore:

$$\begin{aligned} P(\mathrm{D} \,\&\, \mathrm{H2}) &= P(\mathrm{H2}) = P(\mathrm{H3}) = P(\mathrm{D} \,\&\, \mathrm{H3}) \\ P(\mathrm{D} \,\&\, \neg\mathrm{H2}) &= P(\neg(\neg\mathrm{D} \vee \mathrm{H2})) = 1 - P(\neg\mathrm{D} \vee \mathrm{H2}) \\ &= 1 - (P(\neg\mathrm{D}) + P(\mathrm{H2})) = 1 - (P(\neg\mathrm{D}) \\ &\quad + P(\mathrm{H3})) = P(\mathrm{D} \,\&\, \neg\mathrm{H3}) \\ P(\neg\mathrm{D} \,\&\, \mathrm{H2}) &= 0 = P(\neg\mathrm{D} \,\&\, \mathrm{H3}) \\ P(\neg\mathrm{D} \,\&\, \neg\mathrm{H2}) &= P(\neg\mathrm{D}) = P(\neg\mathrm{D} \,\&\, \neg\mathrm{H3}) \end{aligned}$$

That is, the joint probability distribution for {D, H2} matches that for {D, H3}, which means that any probability relating to the former pair is identical with the corresponding probability relating to the latter. An account which allows a variation in explanation only if there is a variation in probability is thus not able to honour H2 with explaining D while denying H3 this status. But if probabilistic theories cannot cope with explanation, they will hardly be able to deal with coherence.

My conclusion is therefore that it is not just the specific accounts of coherence discussed here which are on the wrong track. It would rather appear that the whole project should be dismissed because probabilistic measures are not sophisticated enough. Probability might be *one* of the aspects which are crucial to coherence, but coherence is a notion too rich to be captured by *nothing but* probabilistic terms.[13]

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NOTES

[1] Lewis's (1946, p. 338) qualitative theory of 'congruence' may be deemed a forerunner of these probabilistic measures.
[2] Cf. Douven and Meijs (2006, sect. 5.1) and Moretti and Akiba (2006, sects. 1f).
[3] For a further objection to Shogenji's measure, see Bovens and Hartmann (2003b, p. 50). Cf. also Siebel and Wolff (2005), where it is shown that his proposal and that of Douven and Meijs do not respect the intuition that equivalent witness reports are highly coherent.
[4] Bovens and Hartmann (2003b, p. 50) point out an additional difficulty.
[5] Although developed independently of Douven and Meijs's (2006, sects. 2f.) recipe for generating probabilistic measures of coherence, Fitelson's proposal follows it.
[6] These additions make sure one obtains maximal values for all of these deductive cases. The Kemeny/Oppenheim measure is undefined if $P(B)$ is 0 or $P(A)$ is 0 or 1.
[7] Cf. Siebel (2004) and, for a similar but more complicated argument, Bovens and Hartmann (2003b, p. 51).
[8] For overviews and the pros and cons of different measures, see Kyburg (1983, sect. IV), Eells and Fitelson (2002) and Fitelson (1999, 2001).
[9] Strictly speaking, the common denominator of probabilistic accounts of confirmation is a slightly qualified variant of the relevance criterion. For Fitelson takes B to support A if A is a logical truth and B not a logical falsehood, although in this case $P(A|\ B) = P(A)$. But if we restrict the relevance criterion to contingent propositions, all probabilists will agree to it.
[10] See Douven and Meijs (2006, sect. 4), for a further problem with Fitelson's measure.
[11] Cf. Shogenji (2001, p. 149f.) and Moretti and Akiba (2006, sect. 8).
[12] Cf. also Thagard (1992, ch. 4.1) and Bartelborth (1996, sects. IV.D, IV.F).
[13] In Siebel (2005), I make a point in favour of a corrected variant of Thagard's measure (cf. Thagard 1992, ch. 4; Thagard and Verbeurgt 1998, sect. 2).

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